

The Raman tensors for all the magnetic point groups

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Since the Raman tensors depend on the choice of the coordinate system, we adopt the lattice symmetry directions of crystallographic systems, as summarized in Table 1. The Raman tensors for all magnetic point groups (MPG) are listed in Table 2-33. These results are also given on GitHub website [<https://ruichun.github.io/MagneticRamanTensor/>], where the character tables and direct product are also given. Please note that:

1. The symmetric part $[\alpha_{ij}^S]$ is time-reversal even (T -even), and satisfies $\alpha^S(-\omega) = [\alpha^S(\omega)]^*$. While the antisymmetric part $[\alpha_{ij}^A]$ is time-reversal odd (T -odd), and satisfies $\alpha^A(-\omega) = [-\alpha^A(\omega)]^*$. Consequently, the T -even $\alpha_{ij}^S(0)$ may have a nonzero real component at zero frequency limit. Whereas $\alpha^A(0)$ vanishes, indicating the α^A belongs a non-equilibrium tensor.
2. The Raman coefficients $\alpha_{ij}(\omega)$ possess both real and imaginary parts, analogous to the dielectric function, and satisfy the Kramers-Kronig relations.
3. The second row of each table presents the symmetric components of the Raman tensor, and the Raman tensor elements are denoted with the letter “ α ”. While the remaining rows correspond to the antisymmetric components, and the Raman tensor elements are written as the letter “ A ”. The symmetric parts are the same as the known results in the textbooks and literature.
4. Since the black-and-white MPG $M = S + R'(G - S)$ has the unitary (R) and anti-unitary (R') elements, we assign them the values +1 and -1, respectively, as the main text. In this way, the MPG M can be regarded as a 1D real irrep χ_{mag} of the isomorphic nonmagnetic point group G , as listed in the first column of each table.
5. For double degenerate E modes and triply degenerate T modes, three indices are used (e.g., α_{112} and A_{212}), where the first index denotes the number of the irreducible representation, and the last two are the Raman tensor subscripts.
6. For the MPGs $42'2'$, $4m'm'$, $\bar{4}2'm'$, $4/mm'm'$, $32'$, $3m'$, $\bar{6}m'2'$, $\bar{3}m'$, $62'2'$, $6m'm'$, $6/mm'm'$, $\bar{4}'3m'$, $4'32'$, and $m\bar{3}m'$, the E modes undergo splitting, forming the chiral phonons (such as $\text{Co}_3\text{Sn}_2\text{S}_2$, and we indicate them by [S] after the MPG names.
7. For the origin point groups 3 , 4 , 6 , $3/m$, $4/m$, $6/m$, $\bar{4}$, $\bar{3}$, 23 , and $m\bar{3}$, the irrep E modes also split, but they become degenerate again in the corresponding grey groups.

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TABLE 1: Lattice symmetry directions of 32 original point groups.

| Crystallographic system | Point Groups | Direction 1 | Direction 2 | Direction 3 |
|-------------------------|---|--------------|---|---|
| Cubic | $23, m\bar{3}, 432, \bar{4}3m, m\bar{3}m$ | a/b/c | body diagonal <i>e.g.</i> a + b + c | face diagonal <i>e.g.</i> a + b |
| Hexagonal | $6, \bar{6}, 6/m, 622, 6mm, \bar{6}m2, 6/mmm$ | c | a/b | a + 2b/2a + b |
| Tetragonal | $4, \bar{4}, 4/m, 422, 4mm, \bar{4}2m, 4/mmm$ | c | a/b | face diagonal <i>e.g.</i> a + b |
| Trigonal | $3, \bar{3}, 32, 3m, \bar{3}m$ | c | a/b | NA |
| Orthorhombic | $222, 2mm, mmm$ | c | a | b |
| Monoclinic | $2, m, 2/m$ | c | NA | NA |
| Triclinic | $1, \bar{1}$ | NA | NA | NA |

1. The letters “NA” (not available) in the third to the fifth column indicate that there are no symmetry operations along these characteristic directions.
2. The symbols “/” in the third to the fifth columns denote that there are two or more equivalent character directions.
3. The character directions for grey MPGs and black-white MPGs obey the rules similar to those in original MPGs.

TABLE 2: The Raman tensors of the point group 1 (C_1) and corresponding MPG.

| 1 (C_1) | A |
|-------------|---|
| Sym. | $\begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{12} & \alpha_{22} & \alpha_{23} \\ \alpha_{13} & \alpha_{23} & \alpha_{33} \end{pmatrix}$ |
| 1 (A) | $\begin{pmatrix} 0 & A_{12} & -A_{31} \\ -A_{12} & 0 & A_{23} \\ A_{31} & -A_{23} & 0 \end{pmatrix}$ |

TABLE 3: The Raman tensors of the point group $\bar{1}$ (C_i) and corresponding MPGs.

| $\bar{1}$ (C_i) | A_g | A_u |
|----------------------|---|--|
| Sym. | $\begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{12} & \alpha_{22} & \alpha_{23} \\ \alpha_{13} & \alpha_{23} & \alpha_{33} \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ |
| $\bar{1}$ (A_g) | $\begin{pmatrix} 0 & A_{12} & -A_{31} \\ -A_{12} & 0 & A_{23} \\ A_{31} & -A_{23} & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ |
| $\bar{1}'$ (A_u) | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & A_{12} & -A_{31} \\ -A_{12} & 0 & A_{23} \\ A_{31} & -A_{23} & 0 \end{pmatrix}$ |

TABLE 4: The Raman tensors of the point group 2 (C_2) and corresponding MPGs.

| 2 (C_2) | A | B |
|-------------|---|---|
| Sym. | $\begin{pmatrix} \alpha_{11} & \alpha_{12} & 0 \\ \alpha_{12} & \alpha_{22} & 0 \\ 0 & 0 & \alpha_{33} \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & \alpha_{13} \\ 0 & 0 & \alpha_{23} \\ \alpha_{13} & \alpha_{23} & 0 \end{pmatrix}$ |
| 2 (A) | $\begin{pmatrix} 0 & A_{12} & 0 \\ -A_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & -A_{31} \\ 0 & 0 & A_{23} \\ A_{31} & -A_{23} & 0 \end{pmatrix}$ |
| 2' (B) | $\begin{pmatrix} 0 & 0 & -A_{31} \\ 0 & 0 & A_{23} \\ A_{31} & -A_{23} & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & A_{12} & 0 \\ -A_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ |

TABLE 13: The Raman tensors of the point group 422 (D_4) and corresponding MPGs.

| 422 (D_4) | A_1 | A_2 | B_1 | B_2 | E |
|---------------------|---|--|--|---|--|
| Sym. | $\begin{pmatrix} \alpha_{11} & 0 & 0 \\ 0 & \alpha_{11} & 0 \\ 0 & 0 & \alpha_{33} \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} \alpha_{11} & 0 & 0 \\ 0 & -\alpha_{11} & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & \alpha_{12} & 0 \\ \alpha_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \alpha_{123} \\ 0 & \alpha_{123} & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & 0 & \alpha_{123} \\ 0 & 0 & 0 \\ \alpha_{123} & 0 & 0 \end{pmatrix}$ |
| 422 (A_1) | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & A_{12} & 0 \\ -A_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & A_{123} \\ 0 & -A_{123} & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & 0 & A_{123} \\ 0 & 0 & 0 \\ -A_{123} & 0 & 0 \end{pmatrix}$ |
| 42'2' (A_2) [S] | $\begin{pmatrix} 0 & A_{12} & 0 \\ -A_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & A_{113} \\ 0 & 0 & 0 \\ -A_{113} & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -A_{113} \\ 0 & A_{113} & 0 \end{pmatrix}$ |
| 4'22' (B_1) | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & A_{12} & 0 \\ -A_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & A_{123} \\ 0 & -A_{123} & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & 0 & -A_{123} \\ 0 & 0 & 0 \\ A_{123} & 0 & 0 \end{pmatrix}$ |
| 4'2'2 (B_2) | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & A_{12} & 0 \\ -A_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & A_{113} \\ 0 & 0 & 0 \\ -A_{113} & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & A_{113} \\ 0 & -A_{113} & 0 \end{pmatrix}$ |

TABLE 14: The Raman tensors of the point group $4mm$ (C_{4v}) and corresponding MPGs.

| $4mm$ (C_{4v}) | A_1 | A_2 | B_1 | B_2 | E |
|-----------------------|---|--|--|---|--|
| Sym. | $\begin{pmatrix} \alpha_{11} & 0 & 0 \\ 0 & \alpha_{11} & 0 \\ 0 & 0 & \alpha_{33} \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} \alpha_{11} & 0 & 0 \\ 0 & -\alpha_{11} & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & \alpha_{12} & 0 \\ \alpha_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & \alpha_{113} \\ 0 & 0 & 0 \\ \alpha_{113} & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\alpha_{113} \\ 0 & -\alpha_{113} & 0 \end{pmatrix}$ |
| $4mm$ (A_1) | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & A_{12} & 0 \\ -A_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & A_{113} \\ 0 & 0 & 0 \\ -A_{113} & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -A_{113} \\ 0 & A_{113} & 0 \end{pmatrix}$ |
| $4m'm'$ [S] (A_2) | $\begin{pmatrix} 0 & A_{12} & 0 \\ -A_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & A_{123} \\ 0 & -A_{123} & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & 0 & A_{123} \\ 0 & 0 & 0 \\ -A_{123} & 0 & 0 \end{pmatrix}$ |
| $4'mm'$ (B_1) | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & A_{12} & 0 \\ -A_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & A_{113} \\ 0 & 0 & 0 \\ -A_{113} & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & A_{113} \\ 0 & -A_{113} & 0 \end{pmatrix}$ |
| $4'm'm$ (B_2) | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & A_{12} & 0 \\ -A_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & A_{123} \\ 0 & -A_{123} & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & 0 & -A_{123} \\ 0 & 0 & 0 \\ A_{123} & 0 & 0 \end{pmatrix}$ |

TABLE 15: The Raman tensors of the point group $\bar{4}2m$ (D_{2d}) and corresponding MPGs.

| $\bar{4}2m$ (D_{2d}) | A_1 | A_2 | B_1 | B_2 | E |
|-----------------------------|---|--|--|---|--|
| Sym. | $\begin{pmatrix} \alpha_{11} & 0 & 0 \\ 0 & \alpha_{11} & 0 \\ 0 & 0 & \alpha_{33} \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} \alpha_{11} & 0 & 0 \\ 0 & -\alpha_{11} & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & \alpha_{12} & 0 \\ \alpha_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \alpha_{123} \\ 0 & \alpha_{123} & 0 \\ 0 & 0 & -\alpha_{123} \\ 0 & 0 & 0 \\ -\alpha_{123} & 0 & 0 \end{pmatrix}$ |
| $\bar{4}2m$ (A_1) | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & A_{12} & 0 \\ -A_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & A_{123} \\ 0 & -A_{123} & 0 \\ 0 & 0 & -A_{123} \\ 0 & 0 & 0 \\ A_{123} & 0 & 0 \end{pmatrix}$ |
| $\bar{4}2'm'$ (A_2) [S] | $\begin{pmatrix} 0 & A_{12} & 0 \\ -A_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & A_{113} \\ 0 & 0 & 0 \\ -A_{113} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & A_{113} \\ 0 & -A_{113} & 0 \end{pmatrix}$ |
| $\bar{4}'2m'$ (B_1) | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & A_{12} & 0 \\ -A_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & A_{123} \\ 0 & -A_{123} & 0 \\ 0 & 0 & A_{123} \\ 0 & 0 & 0 \\ -A_{123} & 0 & 0 \end{pmatrix}$ |
| $\bar{4}'2'm$ (B_2) | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & A_{12} & 0 \\ -A_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & A_{113} \\ 0 & 0 & 0 \\ -A_{113} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -A_{113} \\ 0 & A_{113} & 0 \end{pmatrix}$ |

TABLE 18: The Raman tensors of the point group $\bar{3} (S_6)$ and corresponding MPGs.

| $\bar{3} (S_6)$ | A_g | A_u | 1E_g | 2E_g | 1E_u | 2E_u |
|------------------|---|--|--|--|---|---|
| Sym. | $\begin{pmatrix} \alpha_{11} & 0 & 0 \\ 0 & \alpha_{11} & 0 \\ 0 & 0 & \alpha_{33} \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} \alpha_{11} & i\alpha_{11} & \alpha_{13} \\ i\alpha_{11} & -\alpha_{11} & -i\alpha_{13} \\ \alpha_{13} & -i\alpha_{13} & 0 \end{pmatrix}$ | $\begin{pmatrix} \alpha_{11} & -i\alpha_{11} & \alpha_{13} \\ -i\alpha_{11} & -\alpha_{11} & i\alpha_{13} \\ \alpha_{13} & i\alpha_{13} & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ |
| $\bar{3} (A_g)$ | $\begin{pmatrix} 0 & A_{12} & 0 \\ -A_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & -A_{31} \\ 0 & 0 & iA_{31} \\ A_{31} & -iA_{31} & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & -A_{31} \\ 0 & 0 & -iA_{31} \\ A_{31} & iA_{31} & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ |
| $\bar{3}' (A_u)$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & A_{12} & 0 \\ -A_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & -A_{31} \\ 0 & 0 & iA_{31} \\ A_{31} & -iA_{31} & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & -A_{31} \\ 0 & 0 & -iA_{31} \\ A_{31} & iA_{31} & 0 \end{pmatrix}$ |

TABLE 19: The Raman tensors of the point group $32 (D_3)$ and corresponding MPGs.

| $32 (D_3)$ | A_1 | A_2 | E | |
|-----------------|---|--|--|---|
| Sym. | $\begin{pmatrix} \alpha_{11} & 0 & 0 \\ 0 & \alpha_{11} & 0 \\ 0 & 0 & \alpha_{33} \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} \alpha_{111} & 0 & \alpha_{113} \\ 0 & -\alpha_{111} & 0 \\ \alpha_{113} & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & -\alpha_{111} & 0 \\ -\alpha_{111} & 0 & \alpha_{113} \\ 0 & \alpha_{113} & 0 \end{pmatrix}$ |
| $32 (A_1)$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & A_{12} & 0 \\ -A_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & A_{113} \\ 0 & 0 & 0 \\ -A_{113} & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & A_{113} \\ 0 & -A_{113} & 0 \end{pmatrix}$ |
| $32' (A_2) [S]$ | $\begin{pmatrix} 0 & A_{12} & 0 \\ -A_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & A_{123} \\ 0 & -A_{123} & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & -A_{123} \\ 0 & 0 & 0 \\ A_{123} & 0 & 0 \end{pmatrix}$ |

TABLE 20: The Raman tensors of the point group $3m (C_{3v})$ and corresponding MPGs.

| $3m (C_{3v})$ | A_1 | A_2 | E | |
|-----------------|---|--|--|---|
| Sym. | $\begin{pmatrix} \alpha_{11} & 0 & 0 \\ 0 & \alpha_{11} & 0 \\ 0 & 0 & \alpha_{33} \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} \alpha_{111} & 0 & 0 \\ 0 & -\alpha_{111} & \alpha_{123} \\ 0 & \alpha_{123} & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & -\alpha_{111} & -\alpha_{123} \\ -\alpha_{111} & 0 & 0 \\ -\alpha_{123} & 0 & 0 \end{pmatrix}$ |
| $3m (A_1)$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & A_{12} & 0 \\ -A_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & A_{123} \\ 0 & -A_{123} & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & -A_{123} \\ 0 & 0 & 0 \\ A_{123} & 0 & 0 \end{pmatrix}$ |
| $3m' (A_2) [S]$ | $\begin{pmatrix} 0 & A_{12} & 0 \\ -A_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & A_{113} \\ 0 & 0 & 0 \\ -A_{113} & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & A_{113} \\ 0 & -A_{113} & 0 \end{pmatrix}$ |

